Laser Physics 5

Inhomogeneous broadening

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Homogeneous vs Inhomogeneous

• Homogeneous broadening: all the atoms have the same behavior, are described by the same cross section $\sigma(\nu)$

\[ \alpha(\nu) = \sigma(\nu) \Delta n \]

• Broadening mechanism: short coherence lifetime (dephasing collisions, phonons, magnetic fields, electric fields, ...)

• Inhomogeneous broadening: the atoms behave slightly differently, resulting in slightly shifted resonances

• This dispersion in resonance leads to an overall broadening of the transition

• If this dispersion is much broader than the homogeneous profile, the width is referred to as “purely inhomogeneous”.
Example: Gaussian inhomogeneous profile

- Statistical nature of the physical origin often (but not always!) leads to a Gaussian profile:

\[
G(\nu - \nu_0) = \frac{2}{\Delta \nu_{\text{inhom}}} \sqrt{\frac{\ln 2}{\pi}} \exp \left[ -\ln 2 \left( \frac{\nu - \nu_0}{\Delta \nu_{\text{inhom}}/2} \right)^2 \right]
\]

- Example: Doppler broadening. Frequency shift due to atom velocity:

\[
\frac{\nu - \nu_0}{\nu_0} = \frac{v_z}{c_0}
\]

- Probability distribution for the velocity component along the \( z \) axis is Gaussian at thermal equilibrium:

\[
P(v_z) dv_z = \left( \frac{m}{2\pi k_B T} \right)^{1/2} \exp \left[ -\frac{mv_z^2}{2k_B T} \right] dv_z
\]
Example: Gaussian inhomogeneous profile

- The probability for the resonance to lie in the interval \([\nu, \nu + \Delta \nu]\) seen along the \(z\) axis is proportional to this velocity distribution:

- This leads to the following normalized profile and “Doppler” linewidth (\(M\) is molar mass, \(R\) is the ideal gas constant):

\[
G_D(\nu - \nu_0) = \frac{c_0}{\nu_0} \sqrt{\frac{m}{2\pi k_B T}} \exp \left[ -\frac{mc_0^2}{2k_B T} \left( \frac{\nu - \nu_0}{\nu_0} \right)^2 \right]
\]

\[
\Delta \nu_D = \frac{\nu_0}{c_0} \sqrt{\frac{8RT \ln 2}{M}},
\]

- Other examples of inhomogeneous broadening: ions in a crystalline host, isotopic mixture, etc...
Unsaturated amplification coefficient

- In the general case, the distribution of resonance frequencies $\nu_i$ denoted as $P(\nu_i - \nu_0)$, centered in $\nu_0$
- Assume pumping is the same for all atom class ($\Delta n_0$ does not depend on $\nu_i$)
- Population inversion for atoms whose resonance frequencies lie in the range $[\nu_i, \nu_i + d\nu_i]$:

$$d\Delta n_i = \Delta n_0 P(\nu_i - \nu_0)d\nu_i$$
Unsaturated amplification coefficient

- For the atom with resonance frequency $\nu_i$, the cross-section $\sigma_i(\nu)$ is given by the normalized homogeneous profile $g(\nu - \nu_i)$

$$\sigma_i(\nu) = \frac{\sigma_0}{g(0)} g(\nu - \nu_i)$$

- Overall unsaturated gain obtained by summing over all classes of atoms

$$\alpha_0(\nu) = \frac{\sigma_0}{g(0)} \Delta n_0 \int_{-\infty}^{\infty} g(\nu - \nu_i) P(\nu_i - \nu_0) d\nu_i$$

![Diagram](image)
Unsaturated amplification coefficient: limiting cases

- In the limiting cases: purely homogeneous or purely inhomogeneous,

\[
\alpha_0(\nu) = \frac{\sigma_0}{g(0)} \Delta n_0 \int_{-\infty}^{\infty} g(\nu - \nu_i) P(\nu_i - \nu_0) d\nu_i
\]

- becomes

\[
\alpha_0(\nu) = \sigma(\nu) \Delta n_0 \quad \text{if} \quad \Delta \nu_{\text{inhom}} \ll \Delta \nu_{\text{hom}} ,
\]

\[
\alpha_0(\nu) = \frac{\sigma_0}{g(0)} P(\nu - \nu_0) \Delta n_0 \quad \text{if} \quad \Delta \nu_{\text{hom}} \ll \Delta \nu_{\text{inhom}}
\]
Spectral hole burning

- Now let us consider saturation of population inversion by a wave at frequency $\nu$. The saturation intensity at frequency $\nu$ for atoms in the class $i$ is

$$I_{\text{sat},i}(\nu) = \frac{h\nu}{2*\sigma_i(\nu)\tau} = I_{\text{sat}0} \frac{g(0)}{g(\nu - \nu_i)}$$

- $I_{\text{sat}0} = h\nu/2*\sigma_0\tau$ is the saturation intensity at resonance. Population inversion for this class of atoms is therefore:

$$d\Delta n_i = P(\nu_i - \nu_0)\Delta n_0 \frac{1}{\frac{I}{I_{\text{sat},i}(\nu)} + \frac{I}{I_{\text{sat}0}}} \frac{1}{d\nu_i}$$

$$= P(\nu_i - \nu_0)\Delta n_0 \left\{ 1 - \frac{I}{I_{\text{sat}0}} + \frac{g(0)}{g(\nu - \nu_i)} \right\} d\nu_i$$
Spectral hole burning

\[ d\Delta n_i = P(\nu_i - \nu_0) \Delta n_0 \left\{ 1 - \frac{I}{I_{sat0}} \frac{1}{\frac{I}{I_{sat0}} + g(0)} - \frac{g(0)}{g(\nu - \nu_i)} \right\} d\nu_i \]

- Saturation term is nonzero only close to resonance
- Burn a hole with relative depth given by

\[ \frac{I}{I_{sat0}} + \frac{g(0)}{g(\nu - \nu_i)} \]
Spectral hole burning for Lorentzian homogeneous profile

- For a Lorentzian homogeneous profile

\[ g(\nu - \nu_i) \equiv L(\nu - \nu_i) = \frac{2}{\pi \Delta \nu_{\text{hom}}} \frac{1}{1 + \left[2(\nu - \nu_i)/\Delta \nu_{\text{hom}}\right]^2} \]

- The saturation term reads

\[ \frac{I}{I_{\text{sat0}}} + \frac{g(0)}{g(\nu - \nu_i)} = \frac{I}{I_{\text{sat0}}} + \frac{1}{1 + \left(\frac{2(\nu - \nu_i)}{\Delta \nu_{\text{hom}} \sqrt{1 + I/I_{\text{sat0}}}}\right)^2} \]

- In the purely inhomogenous approximation, this leads to a hole of width

\[ \Delta \nu' = \Delta \nu_{\text{hom}} \sqrt{1 + \frac{I}{I_{\text{sat0}}}} \]

- Only the atoms close to resonance participate, the others are passive spectators

Saturation broadening
Saturated amplification coefficient

- Gain is obtained by summing over all classes of atoms
  \[ \alpha(\nu) = \int_{-\infty}^{+\infty} \sigma_i(\nu) d\Delta n_i \]

- Leading to
  \[ \alpha(\nu) = \sigma_0 \Delta n_0 \int_{-\infty}^{+\infty} d\nu_i P(\nu_i - \nu_0) \frac{1}{g(0) + \frac{I}{I_{sat0}}} + \frac{I}{I_{sat0}} \]

- In the purely inhomogeneous case with \( P = G \) and \( g = L \), one can assume \( P(\nu_i - \nu_0) \approx P(\nu - \nu_0) \), and take it out of the integral

\[ \alpha(\nu) = \frac{\alpha_0(\nu)}{\sqrt{1 + \frac{I}{I_{sat0}}}} = \frac{\alpha_0(\nu_0)}{\sqrt{1 + \frac{I}{I_{sat0}}}} \frac{P(\nu - \nu_0)}{P(0)} \]

- Saturation reproduces the inhomogeneous profile, but reduces the gain
In a mostly inhomogeneously broadened gain medium, several longitudinal modes may oscillate simultaneously.

Indeed each mode oscillating at $v_q$ corresponds to a class of atoms that interact only with this mode if the homogeneous width is much smaller than the FSR of the laser.

Multiple spectral holes are burnt in the population inversion.

Laser operation in purely inhomogenous case
Laser operation in purely inhomogenous case

- If $I_q$ is the intensity of mode $q$, we write the equality of gain and losses for each mode

$$\alpha(\nu_q) = \frac{\alpha_0(\nu_q)}{\sqrt{1 + I_q/I_{sat0}}} = \frac{\Pi}{L_a}$$

- Leading to the following expression for $I_q$

$$I_q = I_{sat0} \left[ \left( \frac{\alpha_0(\nu_0)L_a}{\Pi} \frac{P(\nu_q - \nu_0)}{P(0)} \right)^2 - 1 \right]$$

- Intensity of the mode varies like the square of the inhomogeneous profile. Output power is

$$I_{out} = T I_{sat0} \left[ \left( \frac{\alpha_0(\nu_q)L_a}{T + \bar{\gamma}} \right)^2 - 1 \right]$$
CW operation of a multimode laser leads to several new physical problems

- **Mode competition**: do the mode oscillate simultaneously, or does one of them oscillate alone?

- **Mode intensities**: what happens to the previous result when the broadening is not purely inhomogeneous

- **Mode phase**: what is the relative phase of simultaneously oscillating modes, and what does it imply?
Spectral hole burning in a linear cavity

For a single-mode gas laser in a linear cavity, the light interacts with two velocity classes

\[ \frac{\Delta v_z}{c} = \frac{\Delta v_h}{\nu} \]

\[ \nu \left( 1 - \frac{v_+}{c_0} \right) = \nu_0 \]

\[ \nu \left( 1 + \frac{v_-}{c_0} \right) = \nu_0 \]
Spectral hole burning in a linear cavity

- The laser extracts energy from two velocity classes
- If \( \nu = \nu_0 \), the two holes merge, and atoms with zero longitudinal velocity interact with both counterpropagating waves
- Neglecting spatial hole burning, the gain is

\[
\begin{align*}
\nu \neq \nu_0 & \Rightarrow \alpha(\nu) = \frac{\alpha_0(\nu)}{\sqrt{1 + I/I_{sat0}}}, \\
\nu = \nu_0 & \Rightarrow \alpha(\nu) = \frac{\alpha_0(\nu)}{\sqrt{1 + 2I/I_{sat0}}}. 
\end{align*}
\]

- As a consequence, when \( \nu = \nu_0 \), the total power decreases by a factor of 2, over a width equal to the homogeneous linewidth of the gain medium
- This is known as the ’Lamb dip’
• The same phenomenon exists in absorption
• In a laser including a saturable absorber, an increase in intensity is observed when the laser frequency coincides with the resonance of the absorbing cell.

• Due to bleaching of the zero velocity class

• Used to stabilize lasers in frequency
Mode selection in a laser can be performed by inserting a filter in the cavity (introducing frequency-dependent losses):

- Birefringent filter (Lyot filter)
- Prism or grating
- Fabry Perot Etalon
- Subcavity acting as filters

Once single mode laser operation is reached, stabilization is required for various applications using an external reference (stabilized cavity, atomic resonance) and an active feedback loop on the laser cavity