Slow-light: Fascinating physics or potential applications?

Slow light using semiconductor optical amplifiers: Model and noise characteristics

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Abstract

We developed an improved model in order to predict the RF behavior of the SOA valid for any experimental conditions. It takes into account the dynamic saturation of the SOA, which can be fully characterized by a simple measurement, and only relies on material fitting parameters, independent of the optical intensity and bias current. We used this new model to analyze and model the additive noise of the SOA in order to fully characterize the influence of the slow light effect on the microwave photonics link properties. To cite this article: P. Berger et al., C. R. Physique 10 (2009).

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1. Introduction

The control of the propagation velocity of optically carried microwave signals has been extensively studied over the past few years. The ability to generate continuously tunable delay lines can indeed now be readily implemented by electronic means. Slow light elements are therefore highly promising in the field of microwave photonics, with applications such as complex filtering of microwave signals, synchronization of optoelectronic oscillators, and control of optically fed phased array antennas [1–3]. Among the different slow light elements, one of the most mature approaches for integration in real field systems relies on Coherent Population Oscillations (CPO) in semiconductor structures [4–6]. This approach indeed offers compactness, continuous tunability of the delay through current injection and possible parallelism [7,8].

Numerous theoretical models describing the slow light effect in semiconductor amplifiers (SOAs) can be found in the literature [6,9–12]. They offer a comprehensive understanding of the slow light phenomena, the gain saturation dynamics and associated group index changes. On the one hand, a few theoretical models rely on a full description of the energy levels of the semiconductor structure [10,13], which are unfortunately usually not available when commercial SOAs are used. On the other hand, the slow light effect in SOAs is well explained by more practical models [6,14], but it is usually assumed that the saturation power and the carrier recombination lifetime are both constant along the propagation direction, by considering effective averaged parameters. These latter models thus become more phenomenological and fail to predict the SOA properties when the input optical power and the injected current vary over a large range.

However, a practical implementation of a semiconductor slow light element in a real microwave photonics link requires an accurate modeling of the impact of the component on the link performance. This includes gain and phase shifts caused by CPO, but also the ability of the link to transmit an analog RF signal without degrading its characteristics, which can be analyzed through the additive intensity noise and the dynamic range. Furthermore, since the generated delays are tuned by controlling the injected current and/or the optical input power, it is essential to have a reliable predictive model that can manage any change in these initial conditions.

In that frame, we propose in this paper an improved model that enables to accurately predict the RF gain compression and the RF phase delay. The adjustment parameters of our model are the material recombination coefficients, which are independent of experimental conditions, thus ensuring the predictive capability of the model for a given component. Furthermore, we show that this model is also efficient in describing the additional relative intensity noise of the SOAs.

2. Slow light properties in SOA

2.1. Model

We consider an optical carrier modulated by an RF signal and injected in a traveling wave SOA. The total field is then composed of the optical carrier of complex amplitude \(E_0\) and two sidebands of complex amplitudes \(E_1\) and \(E_2\). Under the small RF signal approximation, we have \(E_0 \gg E_1, E_2\), and the optical power inside the SOA can thus be written as \(P = |E_{total}|^2 \times S_{guide} = P_0 + M e^{-i\Omega t} + \text{c.c.}\), where \(P_0\) is the DC component of the power, and \(M = (E_0 E_0^* + E_1 E_1^*) S_{guide}\) is the beat-note term at the RF frequency \(\Omega\). \(S_{guide}\) denotes the modal area of the SOA.

The local equations for the propagation of the optical fields and the evolution of carrier density inside the SOA lead to, for a small trench of SOA of thickness \(d\zeta\) [9]:

\[
\frac{dP_0}{d\zeta} = P_0 \left[ -\gamma + \Gamma g(\bar{N}) \right] 
\]

\[
\frac{dM}{d\zeta} = M \left[ -\gamma + \Gamma g(\bar{N}) \left( 1 - \frac{P_0/P_s(\bar{N})}{1 + P_0/P_s(\bar{N}) - i\Omega \tau_s(\bar{N})} \right) \right] 
\]

where \(\bar{N}\) is the steady state carrier density, \(\gamma\) holds for the internal losses of the SOA, and \(\Gamma g(\bar{N})\) is the material modal gain. \(\tau_s(\bar{N})\) is the carrier lifetime and \(P_s(\bar{N})\) is the saturation power defined as: \(P_s(\bar{N}) = \frac{\hbar \omega}{a(\bar{N}) \tau_s(\bar{N})} S_{guide}\), where \(a\) is the differential gain \(a = \frac{\partial g}{\partial N}|_{\bar{N}}\).
The common approach to solve Eqs. (1) and (2) is to consider \(a, \tau_s\), and \(P_s\) constant with respect to the carrier density and thus over the whole length of the device [6]. Doing so, the system of equations can be analytically solved. However, in this case, the adjustment parameters \(\tau_s\) and \(P_s\) vary with experimental conditions, and cannot be a priori estimated: the model thus becomes more phenomenological than predictive. Our focus in this article is to keep the predictive capability of the model. Consequently, we take into account the variations of \(a, \tau_s\), and therefore \(P_s\), with respect to the carrier density modifications along the propagation axis. Our central hypothesis is that \(a\) and \(\tau_s\) can be determined as functions of the DC component of the local optical power \(P_0\) solely, allowing these dependencies to be determined from gain measurements.

The first step is to determine the variations of the modal gain \(\Gamma g\) and of \(\bar{N}/\tau_s\) as a function of the local DC optical power \(P_0\). Consequently, a small signal gain measurement is conducted. It is understood by optical small signal that the injected optical power is lower than the SOA saturation optical power. If small signal conditions are fulfilled, the stimulated emission is negligible compared to the spontaneous emission, leading to the unsaturated steady state solution of the rate equation for the carriers:

\[
\frac{I}{qL S_{act}} = \frac{\bar{N}}{\tau_s}
\]  

(3)

where \(L\) is the length of the SOA, \(S_{act}\) is the area of the active section of the SOA, and \(I\) the injected current. Moreover, if we also assume in this case that the carrier density \(\bar{N}\) is constant along the SOA, a measurement of the small signal modal gain \(\Gamma g_0\) versus \(I\) will be equivalent, owing to Eq. (3), to a determination of the modal gain \(\Gamma g\) versus \(\bar{N}/\tau_s\). Here, \(\Gamma\) is the ratio \(S_{act}/S_{guide}\) of the active to modal gain areas in the SOA.

Combining this result with the saturated steady state solution of the carriers rate equation, one obtains the following set of equations:

\[
\frac{I}{qL} - \frac{\bar{N}}{\tau_s} - \frac{\Gamma g(\bar{N})}{\hbar \omega} P_0 = 0
\]

\[
\Gamma g = \Gamma g(\bar{N}/\tau_s)
\]

(4)

By solving this set of equations, one obtains the variations of \(\Gamma g\) and \(\bar{N}/\tau_s\) versus \(P_0\) and \(I\), i.e., as a function of experimental parameters.

In order to get the differential gain and the carrier lifetime as well, an additional relation that enables to express \(\bar{N}\) and \(\tau_s\) as a function of \(\bar{N}/\tau_s\) and \(P\) is required. This last relation is given by the well-known equation which establishes the carrier lifetime dependence of our SOA versus carrier density [15]:

\[
\frac{1}{\tau_s} = A + B \bar{N} + C \bar{N}^2
\]

(5)

where \(A, B,\) and \(C\), which are, respectively, the non-radiative, spontaneous and Auger recombination coefficients, are the only parameters that will have to be fitted from the experimental results.

Using Eq. (5) and the fact that we have proved that \(\bar{N}/\tau_s\) and \(\Gamma g\) can be considered as functions of \(P_0\) only, we can finally obtain \(\bar{N}\), \(\Gamma a = \Gamma g \frac{\partial g}{\partial \bar{N}}\), and \(P_s = \frac{\hbar \omega}{\Delta \tau_s} S_{guide}\) as functions of \(P_0\). This permits one to replace Eqs. (1) and (2) by the following pair of equations:

\[
\frac{dP_0}{dz} = P_0\left[-\gamma + \Gamma g(P_0)\right]
\]

(6)

\[
\frac{dM}{dz} = M\left[-\gamma + \Gamma g(P_0)\left[1 - \frac{P_0/P_s(P_0)}{1 + P_0/P_s(P_0) - i\Omega \tau_s(P_0)}\right]\right]
\]

(7)

Eqs. (6) and (7) are then numerically solved and the microwave transfer function of the SOA, \(S_{21} = \gamma_t \frac{\Gamma a(L)}{\Gamma a(0)}\), where \(\gamma_t\) are the insertion losses, is computed. The initial conditions are: \(M(0) = \sqrt{\gamma_t} \frac{m P_m}{2}\), and \(P_0(0) = \sqrt{\gamma_t} P_m\), with \(P_m\) the DC optical input power, and \(m\) the input modulation rate.

It is important to note that the recombination coefficients \(A, B\) and \(C\) are the only fitting parameters of our model. Once obtained from experimental data, since they are material constants, they are fixed for any other experimental conditions. The validity of this assumptions will be experimentally verified below with a commercially available SOA (InP/InGaAsP Quantum Well Booster Amplifier from COVEGA).
2.2. Experimental determination of the material modal gain $\Gamma g(P_0)$

The preliminary step consists in measuring the unsaturated gain $\Gamma g_0$ for different injected currents, in order to fully characterize the local gain response of the SOA through $\Gamma g(P_0)$ as described in Section 2.1. To get rid of the amplified spontaneous emission, the optical small signal gain is obtained by measuring the SOA unsaturated RF gain $G_{RF}(=|S_{21}|^2)$ at low input optical power (typically 100 µW). In order to avoid possible RF gain saturation induced by coherent population oscillations, this measurement is conducted at 20 GHz RF-frequency, namely, well above $1/\tau_s$. Under these conditions, the optical small signal gain $\Gamma g_0$ is expressed as:

$$\exp(\Gamma g_0 L) = \frac{\sqrt{G_{RF}}}{\gamma L} \exp(-\gamma L)$$

The total losses of the SOA $\gamma L$ are obtained by an additional experimental measurement ($-16.4$ dB in our case). The unsaturated gain of our SOA is displayed in Fig. 1a. From this simple measurement, the material modal gain $\Gamma g$ is then known as a function of the local optical power $P_0(z)$ inside the SOA (Fig. 1b, in solid line).

In Fig. 1b, we also represent, in dashed lines, the material modal gain according to most practical models: $\Gamma g = \frac{g_0}{1 + P/P_s}$, with $g_0$ and $P_s$ constant. This last expression is a straightforward consequence of the assumption that the gain varies with the carrier density according to $g(N) = a(N - N_0)$, with $a$ constant. It can be noticed that a least squares adjustment accurately matches the experimentally derived data only for a reduced range of optical powers (for instance, at low input optical power). However, for the whole power range, only an approximative adjustment can be obtained. As a consequence, one foresees that those practical models will have an accuracy range restricted to limited optical power variations, and therefore limited experimental conditions (device gain, saturation and carrier density levels). In particular, this kind of model might be more adapted to bulk SOAs or electro-absorption modulators, with not too strong carrier density, and hence gain. But they cannot predict the material modal gain for any initial conditions, when the local power varies too much, as it is the case in high carrier confinement quantum well or quantum dots structures. Our model, based on this simple experimental determination of the material modal gain, will fill in this gap, as we will show in the next section.

2.3. Measurement of RF transfer function of the SOA — Comparison with theory

The complex RF transfer function $S_{21}$ of the SOA is measured using a Vector Network Analyzer (VNA) (set-up in Fig. 2). The corresponding RF gain, $20 \log|S_{21}|$ and RF phase shift, arg($S_{21}$) are reported in Fig. 3, where the experimental data are plotted as a function of the modulation frequency $\Omega$. In (a) and (b) are first plotted the experimental measurements for different injected currents $I$, whereas in (c) and (d) the input optical power is changed.
Fig. 2. Schematic representation of the experimental set-up. A laser is externally modulated by a Mach–Zehnder modulator (MZ). The input optical power $P_{in}$ is controlled through a variable optical attenuator. Two optical isolators are inserted at the input and output of the SOA in order to avoid any back-reflection. The photodetector (PD) converts the optical modulation into RF signal. The Vector Network Analyzer (VNA) is calibrated using, as a reference, the optical link without the SOA. The SOA is then inserted into the link and its RF transfer function is measured.

Fig. 2. Montage expérimental. Un modulateur externe permet de moduler l’intensité par le signal RF généré par le port 1 de l’analyseur de réseau (VNA). Des isolateurs sont placés avant et après le SOA. La puissance optique d’entrée $P_{in}$ est contrôlée par un atténuateur variable. Le signal RF, restitué par la photodiode, est ensuite analysé par le VNA (port 2). Le VNA est calibré avec la liaison sans le SOA, afin de ne mesurer que la fonction de transfert du SOA.

In Fig. 3 are also represented the simulation results in dashed lines. The best fit values for the recombination coefficients are: $A = 2 \times 10^9 \text{ s}^{-1}$, $B = 2 \times 10^{-10} \text{ cm}^3 \text{s}^{-1}$, $C = 5 \times 10^{-29} \text{ cm}^6 \text{s}^{-1}$. These values are in the range of those commonly found in the literature for InP/InGaAsP QW structures, the constitutive materials of our device [13,16,17]. The computed complex transfer function shows a very good agreement with the experimental data, either when the injection current or the input optical power is changed, which therefore proves the predictive capability of our model.

Consequently, we showed that the simple measurement of the small signal gain $\Gamma g_0$ versus the injected current, enables to fully characterize the saturation dynamics for any injected current or optical power. The adjustable param-
eters are then reduced to the recombination coefficients $A$, $B$, $C$, which only depend on the material nature and/or characteristics. Once they are determined for a given SOA, we obtain a model which can accurately predict the RF complex transfer function of the SOA, for any experimental conditions.

Fig. 4 reproduces the evolution of the carrier density $N$ along the SOA, and the subsequent variations of the saturation parameters $P_s$, $\tau_s$, and $\Gamma a$. We find nearly one order of magnitude of variation for almost all these parameters, which are, nevertheless, all taken constant in previous practical models. Once again, according to (5), this approximation can be justified in the case of relatively low values of $N$, that is for a not too strong carrier confinement such as in bulk SOAs or electro-absorption modulators. However, this also reinforces the conclusion shown in this paper, that in the case of quantum well or quantum dots structures, it is necessary to take into account the saturation dynamics along the propagation to ensure the accuracy of the model and its robustness versus changes in experimental conditions.

At this stage, the present work thus enables to accurately predict the RF gain and phase shift produced by a semiconductor slow light element when injected current and input power are changed. It therefore gives the functional behavior of this element when integrated in a microwave photonics architecture. However, towards a real integration, it is also required to predict the impact of this device on the link performances. Consequently, in the next section, we will use our model of the RF transfer function to analyze the influence of slow light effect on noise properties and derive a practical predictive model for the relative intensity noise of the SOA.

3. Influence of slow light effect on noise properties

3.1. Model

In order to analyze the influence of slow light effect on the dynamic range of microwave-photonics links, the additional noise in SOAs has to be characterized. The noise is composed of the shot noise, the thermal noise and the relative intensity noise (RIN). The noise spectral density can then be written as:

$$\text{DSP}_{\text{noise}} = RB_e \left( \text{RIN} I_{\text{ph}}^2 + 2eI_{\text{ph}} + \frac{kT}{R} \right)$$

(8)

with RIN the relative intensity noise, $I_{\text{ph}}$ the photodetected current, $B_e$ the electrical bandwidth, $R$ the photodetector load resistance, and $T$ the temperature. The RIN has a specific behavior for SOAs, especially in slow light regime, which we propose to analyze in the following.

The relative intensity noise (RIN) is defined as:

$$\text{RIN} = \frac{\langle \Delta I_{\text{ph}}^2 \rangle}{I_{\text{ph}}^2}$$

(9)
where $\langle \rangle$ denotes time averaging. $I_{\text{ph}}$ is the current of the photodetector, which can be expressed as a function of the total field $E_{\text{total}}$ detected by the photodetector:

$$I_{\text{ph}} = \frac{e}{h \nu} \langle E_{\text{total}}^2 \rangle$$

where this time $\langle \rangle$ denotes an averaging on optical frequencies.

Our approach corresponds to the semi-classical beating theory. The output field of the SOA, $E_{\text{total}}$, is composed of the amplified input signal and the amplified spontaneous emission. Thus the noise density has two contributions, named as signal-spontaneous beat-note and spontaneous–spontaneous beat-note:

$$\langle \Delta I_{\text{ph}}^2 \rangle = S_{\text{ASE-ASE}} + S_{\text{s-ASE}}$$

where $S_{\text{ASE-ASE}}$ and $S_{\text{s-ASE}}$ are the spectral densities originating from the signal-spontaneous beat-note and the spontaneous–spontaneous beat-note, respectively.

In order to express these two contributions, we define the input spontaneous emission power density as the quantum noise source at the input of SOA [18]:

$$\rho_{\text{noise}}^{\text{in}} = \frac{h \nu}{2} \left( F - \frac{1}{G} \right)$$

with $F$ the noise factor, and $G$ the optical gain of the SOA.

Hence, the fields at the input of the SOA are the signal and the input quantum noise. The input intensity is composed of: (1) a signal-spontaneous beat-note, which can be considered as a sum of modulation components at the frequencies $\Omega$: the response of the SOA is the microwave saturated gain of the SOA $|S_{21}(\Omega)|$; and (2) a spontaneous–spontaneous beat-note which is only responsive to the optical gain $G$.

Consequently, the noise output spectral densities are [19]:

$$S_{\text{s-ASE}}(\Omega) = 4 |S_{21}(\Omega)|^2 \rho_{\text{in}}^{\text{noise}} P_{\text{in}}^{\text{sig}} B_e$$

$$S_{\text{ASE-ASE}}(\Omega) = 2 G^2 \left( \rho_{\text{in}}^{\text{noise}} \right)^2 B_0 (1 - \Omega / B_0) B_e$$

with $P_{\text{in}}^{\text{sig}}$ the input signal power in the SOA, $B_0$ the optical bandwidth before the photodetector, $B_e$ the electrical bandwidth of the detection chain. Over the usual range of the microwave signal ($\ll 100$ GHz), $S_{\text{ASE-ASE}}$ is independent of the frequency $\Omega$. Consequently, only the s-ASE beat-note contributes to the frequency dependence of the noise output spectral densities, through $|S_{21}(\Omega)|^2$: the RIN is thus strongly linked to the slow light effect.

3.2. Experiment — Comparison with theory

The first step consists in measuring the noise factor $F$ of the SOA in order to define $\rho_{\text{in}}^{\text{noise}}$. We used an optical method described in [18] to obtain the experimental data displayed in Fig. 5. The empirical fit of these data and the previous modeling of $S_{21}$ developed in Section 2 are the only inputs of the RIN model. No additional parameters or adjustments are necessary.

The specific RIN of the SOA is measured thanks to the method described in [20]. The experimental data are displayed in Fig. 6. The simulations, also reported in Fig. 6, are in a very good agreement with measurements, once
again with no additional adjustment. One can notice that according to (13), the dip experienced in the gain transfer function in the slow light regime, is reported on the RIN. This result forebodes that the dynamic range will be preserved even at low frequencies, where the slow light effect occurs. The predictive model of the RIN developed here will enable us to accurately study further the influence of the integration of SOAs on the dynamic range of the link.

4. Conclusion

We developed an improved model in order to predict the RF behavior and slow light properties of the SOA, valid for any experimental conditions (input optical power, injected current). It takes into account the dynamic saturation along the SOA, fully characterized by the simple measurement of the small signal gain, and only relies on material fitting parameters, independent of the optical intensity and bias current. We showed a remarkably good agreement between the model and the experimental data.

The ease of use and the accurate prediction obtained for any experimental conditions make this model a useful tool to characterize the effect of slow light in SOA on a microwave link. We already shown that it enables to fully describe the additive noise of the SOA. A future study will lead us to study the linear dynamic range of the SOA. Furthermore, in order to determine the harmonic generation, intermodulation products and spurious free dynamic range, a generalization of our approach (relying on the same experimental determination of the material modal gain) will be carried out in a next step, for a full characterization of a SOA based optoelectronic link.

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